Written Exam Economics summer 2016

Industrial Organization

June 6, 2016

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

This exam paper consists of three pages in total, including this one

Attempt both questions.
Explain all the steps of your analysis and define any new notation that you use.
Show all the calculations that your analysis relies on.

Question 1: Persuasive advertising and competition

Consider a market in which $n \geq 2$ firms sell their goods and compete in quantities. The firms interact just once and they make their output decisions simultaneously. Their objective is to maximize the own profit. Inverse demand is a function of the firms' total output, but the intercepts $a_i > 0$ may differ across firms:

$$p_i = a_i - \sum_{j=1}^{n} q_j,$$

where p_i and q_i are firm i's price and output, respectively. All firms have the same constant marginal cost c (with $a_i > c \geq 0$ for all i) and no fixed costs. Therefore firm i's profit can be written as

$$\pi_i = \left[a_i - c - \sum_{j=1}^{n} q_j \right] q_i.$$

Denote the Nash equilibrium outputs by $(q_1^*, \ldots, q_n^*)$.

(a) Assume that the parameters of this model are such that, at the equilibrium, all firms are active (i.e., $q_i^* > 0$ for all i). Show that firm i's equilibrium output is given by

$$q_i^* = \frac{n a_i - \sum_{j \neq i} a_j - c}{n+1}.$$

Now extend the game described above as follows. Assume that the demand intercept is given by

$$a_i \equiv \overline{a} + x_i,$$

where $\overline{a} > 0$ is an exogenous parameter and $x_i \geq 0$ is firm i's choice variable. The variable x_i can be interpreted as the extent to which firm i engages in persuasive advertising: A larger x_i makes the consumers willing to pay more for firm i's good, which is reflected in a larger demand intercept. Choosing a larger x_i is associated with a cost. The cost function is given by

$$\varphi(x_i) = x_i^3,$$

The timing of the game is as follows:

1. The n firms simultaneously choose their advertising levels x_i.
2. The firms observe all chosen advertising levels. Then they simultaneously choose their outputs q_i.

The firms' objective is to maximize their overall profit, denoted by Π_i, where the cost function $\varphi(x_i)$ enters additively. That is,

$$\Pi_i = \left[\overline{a} - c + x_i - \sum_{j=1}^{n} q_j \right] q_i - x_i^3.$$

Denote the subgame-perfect Nash equilibrium levels of advertising by $(x_1^*, \ldots, x_n^*)$.

The firms' interaction at stage 2, for given values of a_i, has already been analyzed in part (a). You should therefore make use of the results stated there when you solve part (b).

(b) Again assume that, at the equilibrium, all firms are active (i.e., $q_i^* > 0$ for all i). Simplify by assuming $\overline{a} - c = 1$ and $n = 2$, and then solve for a symmetric equilibrium value of the advertising levels, x^*. Assume that the second-order condition is satisfied.

- *Hint: The roots of the quadratic equation $Ax^2 + Bx + C = 0$ are given by*

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}.$$

(c) Answer the following questions briefly and in words only:

(i) Why is it problematic to analyze the welfare consequences for consumers of persuasive advertising?

(ii) In the course we also studied another way of thinking about advertising. What was the idea behind that approach?

(iii) When modeling markets with network goods, we showed that there can co-exist multiple demand functions (for a given set of consumer preferences). What is the logic behind that result?

Question 2: Collusion with fluctuating demand

Consider the following version of the Rotemberg-Saloner model. In a market there are n ex ante identical firms, indexed by i. They produce a homogeneous good and each firm has a constant marginal cost $c \geq 0$. There are infinitely many, discrete time periods t (so $t = 1, 2, 3, \ldots$), and at each t the firms simultaneously choose their respective price, p_i^t. The firms' common discount factor is denoted by $\delta \in (0, 1)$. As the good is homogeneous, demand is a function of the lowest price, $p^t = \min\{p_1^t, p_2^t, \ldots, p_n^t\}$. Demand is stochastic: with probability $\lambda \in (0, 1)$, demand in period t is high, $q^t = D_H(p^t)$; and with probability $1 - \lambda$, demand in period t is low, $q^t = D_L(p^t)$, with $D_H(p^t) > D_L(p^t)$ for all p^t. Demand realizations are independent across time. If two or more firms charge the same price, then these firms share the demand equally between themselves.

The firms can observe all rival firms' choice of price once it has been made. Moreover, the firms can observe the current period's demand realization before choosing their price. However, the demand realizations in future periods are not known to the firms.

Let p_s^m be the state $s \in \{L, H\}$ monopoly price, i.e., the price that maximizes $(p - c) D_s(p)$. Exactly as in the course, consider a grim trigger strategy in which each firm starts out charging the price $p_s^t = p_s^m$ if the period t state is s. However, if there has been any deviation from that behavior by anyone of the firms in any previous period, then each firm plays $p_s^t = c$.

(a) Derive a necessary and sufficient condition for when the above grim trigger strategy is part of a subgame-perfect Nash equilibrium.

- State the condition so that δ is isolated on one side of the inequality—i.e., as $\delta \geq \delta_0$, where δ_0 is a function of n, λ and the maximized industry profits in each of the two states, but not a function of δ.
- *Hint: You may want to use the formula for an infinite geometric series:* $\sum_{t=0}^{\infty} \delta^t = 1/(1-\delta)$ *for* $\delta \in (0, 1)$.

(b) When is full collusion most difficult to sustain—in a high or in a low state? Explain the intuition. Answer verbally only.

- *You are encouraged to answer this question even if having failed to solve question (a).*

(c) What is meant by limit pricing and predatory pricing? How did Milgrom and Roberts (in Tirole's simplified version) model limit pricing? Focus on the key model assumptions and explain how the logic of the model works.

End of Exam